A SIMPLE AND GENERAL METHOD FOR DETECTING STRUCTURAL INCONSISTENCIES IN LARGE ELECTRICAL NETWORKS

Gunther Reißig[*]

M.I.T., 77 Massachusetts Av., Room 66-365
Cambridge, MA 02139, U.S.A.
gr@ieee.org

Uwe Feldmann

Infineon Technologies, MP PTS
D-81730 München, Germany
uwe.feldmann@infineon.com

ABSTRACT

We propose to determine fundamental circuits of two matroids induced by certain bipartite graphs derived from the network equations in order to detect inconsistencies in large electrical networks. Our method is easy to implement, is very fast, and does not rely on a specific form of network equations. It has been implemented in the circuit simulator TITAN of Infineon Technologies and, despite its simplicity, is of great help in locating inconsistencies in erroneous network descriptions, especially if these descriptions contain VHDL-AMS code.

1. BACKGROUND

The simulation of an electrical network is usually based on its network equations

$$F(x, \dot{x}) = f(t), \tag{1}$$

which are automatically generated from a description of the electrical network such as a SPICE netlist or VHDL-AMS source code by the simulation software [1]. Generating an appropriate description, however, often requires human interaction and is a difficult and error prone task.

Network equations obtained from an erroneous description might still be simulated, but some of the numerically calculated voltages and currents will be implausible. Usually, by looking at these implausible quantities, the user is able to correct the description of the circuit.

Often, however, the network equations cannot be simulated as they contain inconsistencies. In particular, these equations may not be uniquely solvable or not uniquely DC-solvable, which may be due simply to human error or to oversimplified modeling and also frequently happens in fault simulation. As a simulation is impossible, the source of the problem is usually located by other means:

Prior to any simulation, present circuit simulators check whether the network graph fulfills certain conditions that are sufficient for the unique solvability of the network equations (1), see [2] and the references cited in [3]. If that test fails, some of the available methods provide circuits and cut-sets of critical network elements, which is of great help in correcting the description of the network.

Alternatively, the method proposed in [4] yields minimum regularizations of linear networks that are generically uniquely solvable. That is, the networks are augmented with a minimum number of additional network elements in order to make them generically uniquely solvable. See also [3] for related results.

There are a couple of methods that investigate properties of the network equations (1) directly [5, 6]. These methods check if certain conditions necessary for regularity of the Jacobians $D_1F(x_0, y_0)$ and $D_1F(x_0, y_0) + sD_2F(x_0, y_0)$, for appropriate $s \in \mathbb{R}$ and x_0, y_0, are met. Otherwise, they provide a list of superfluous rows and columns of these Jacobians that linearly depend on the remaining rows and columns. These methods apply graph-theoretical algorithms to bipartite graphs derived from the zero-nonzero structure of the Jacobians. Hence, the necessary conditions checked involve the zero-nonzero structure of the Jacobians only, which is why these methods are called *structural methods*. (For a variant using representation graphs rather than bipartite graphs, and for a generalization of the structural approach, see [7] and the references therein.)

Another simple heuristic is to check certain relations between the number of variables and the number of equations of high level descriptions of network elements as done in VHDL-AMS [8, 9].

While the method from [4] is computationally unacceptably expensive, the methods from both [4] and [2, 3] heavily rely on the network graph and a certain, fixed set of network elements. As a consequence, these methods fail if the network or any part thereof is described in some high level description language.

To check the numbers of variables and equations in the description of the network elements as in VHDL-AMS would point to erroneous element descriptions only, but would not detect any inconsistencies that involve the topology of the network, such as a circuit of voltage sources.

Finally, the structural methods from [5, 6] provide rather incomplete information as they do not reveal on which rows and columns the superfluous rows and columns actually depend.

In conclusion, for networks that contain parts described in some high level language, the current methods for checking for inconsistencies in network descriptions are either not applicable or do not provide as detailed information on how to remove these inconsistencies as is available from the established methods for conventional networks not containing high level language code. This is a serious drawback since the more general the description language is, the more likely is human error and inappropriate modeling. In fact, most of the erroneous network descriptions sent to the hotline of the network simulator TITAN of Infineon Technologies contain VHDL-AMS code.

In this paper, we propose a structural method for detecting inconsistencies in equation (1) which is easy to implement, is very fast, and does not rely on a specific form of (1).

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That method refines those from [5, 6] in that it does not only yield a list of superfluous rows and columns of the Jacobians $D_1F(x_0, y_0)$ and $D_1F(x_0, y_0) + sD_2F(x_0, y_0)$, but also determines on which rows and columns those superfluous rows and columns actually depend. Mathematically, we do not only calculate a maximum matching in a bipartite graph derived from (1), but also determine the fundamental circuits in two matroids induced by that graph with respect to the matching.

Our method is described in section 2, which also contains a bound on its computational complexity. Its application is demonstrated in section 3.

2. DETECTING STRUCTURAL INCONSISTENCIES

To check equation (1) for unique solvability and unique DC-solvability, we assume $F\colon \mathbb{R}^m \times \mathbb{R}^m \to \mathbb{R}^n$ and $f\colon \mathbb{R} \to \mathbb{R}^n$ and test if certain conditions necessary for the regularity of the Jacobians $D_1F(x_0, y_0)$ and $D_1F(x_0, y_0) + sD_2F(x_0, y_0)$ are met.

In a **first step**, as done in all structural methods, we set up an $n \times m$ matrix L. To check for unique solvability, L is defined by

$$L_{i,j} = \begin{cases} 1 & \text{if } F(x, y) \text{ depends on } x_j \text{ or } y_j \\ 0 & \text{otherwise} \end{cases}, \tag{2}$$

for unique DC-solvability, by

$$L_{i,j} = \begin{cases} 1 & \text{if } F(x, y) \text{ depends on } x_j \\ 0 & \text{otherwise} \end{cases}.$$

In a **second step**, we determine a maximum matching in the bipartite graph $\mathcal{G}(L)$ of L, $\mathcal{G}(L) = (V_1, V_2, E)$.

In a **third step**, if the maximum matching found is not perfect, we determine the fundamental circuits of the row matroid of $\mathcal{G}(L)$ with respect to the base $V_1 \cap \partial M$ by growing an Hungarian forest (V_1', V_2', E') in the subgraph of $\mathcal{G}(L)$ induced by the set $(\partial M \cap V_1) \cup V_2 \cup \{v\}$ of vertices, for each $v \in V_1 \setminus \partial M$. The following result shows that, for each such v, the vertex set V_1' of the forest is the fundamental circuit containing v in the row matroid $\mathcal{R}(\mathcal{G}(L))$:

2.1 Theorem *Let G be a bipartite graph, $G = (V_1, V_2, E)$, M a matching in G, and F a maximal M-alternating forest in G, $F = (V_1', V_2', E')$, $V_1' \subseteq V_1$, $V_2' \subseteq V_2$. Then the following holds for any $v \in V_1$: There is a circuit of the row matroid $\mathcal{R}(G)$ containing v iff $v \in V_1'$.*

In a **fourth step**, we determine the fundamental circuits of the column matroid of $\mathcal{G}(L)$ by applying the third step to $\mathcal{G}(L^T)$.

The fundamental circuits in the row (column) matroid of $\mathcal{G}(L)$ correspond to linearly dependent sets of rows (columns) of the Jacobians $D_1F(x_0, y_0)$ and $D_1F(x_0, y_0) + sD_2F(x_0, y_0)$, respectively. Of course, as the above algorithm does not take into account any numerical values of the elements in those Jacobians, the linearly dependent sets of rows (columns) may not be minimal. However, the following can be shown if the nonzero entries of the above Jacobians are independently varying parameters: For all but exceptional numerical values for those parameters, any proper subset of any set of rows (columns) corresponding to a fundamental circuit in the row (column) matroid of $\mathcal{G}(L)$ is linearly independent.

Regarding run time, note that the first step is computationally trivial and takes $O(r)$ operations, where r is the number of nonzeros in the matrix L from (2). A maximum matching is found in the second step in $O(r(n+m)^{1/2})$ operations [10]. It is important to note that augmenting path algorithms for the maximum matching problem actually grow Hungarian forests, and each such forest in the third and fourth steps can be found in $O(r)$ operations, so that the total number of operations of the method is $O(r(n+m))$.

Finally, it should be noted that, as an alternative to the first step above, it is sometimes useful to define the matrix L by the requirement that it has exactly the same zero-nonzero structure as the Jacobians $D_1F(x_0, y_0)$ and $D_1F(x_0, y_0) + sD_2F(x_0, y_0)$, respectively, for appropriate $s \in \mathbb{R}$ and $x_0, y_0 \in \mathbb{R}^m$.

3. EXAMPLES

The following examples are worked out based on the branch-voltage-branch-current equations [3], but any other type of circuit equations could have been chosen instead.

3.1 Example Consider the electrical network from Fig. 1(a). Its network equations are of the form

$$A\dot{x} + Bx = f(t), \tag{3}$$

where $x = (v_R, v_{C_1}, v_{C_2}, i_R, i_{C_1}, i_{C_2})$, v_j and i_j are the voltage across and the current through the network element j,

$$sA + B = \begin{pmatrix} 1 & -1 & 1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & -1 & -1 & \cdot \\ \cdot & \cdot & \cdot & 1 & \cdot & -1 \\ -1 & \cdot & \cdot & R & \cdot & \cdot \\ \cdot & sC_1 & \cdot & \cdot & -1 & \cdot \\ \cdot & \cdot & sC_2 & \cdot & \cdot & -1 \end{pmatrix},$$

and dots $(\cdot)$ denote zero entries.

In order to check for unique solvability, set up the matrix L first: $L_{i,j} = 1$ if $(sA+B)_{i,j} \neq 0$ and $L_{i,j} = 0$ otherwise. The bipartite graph $\mathcal{G}(L)$ is (V_1, V_2, E), where $V_1 = \{1, \ldots, 6\}$, $V_2 = \{7, \ldots, 12\}$, and

$$E = \{\{1,7\},\{1,8\},\{1,9\},\{2,10\},\{2,11\},\{3,10\},\{3,12\},\{4,7\},\{4,10\},\{5,8\},\{5,11\},\{6,9\},\{6,12\}\},$$

see Fig. 1(b). In the second step, the perfect matching

$$\{\{1,7\},\{2,11\},\{3,12\},\{4,10\},\{5,8\},\{6,9\}\}$$

is obtained, i.e., no inconsistencies are found. Indeed, for almost all values of the parameters R, C_1, C_2 and s, the matrix $sA + B$ is regular, and hence, (3) is uniquely solvable.

In order to check for unique DC-solvability, set up the matrix L first: $L_{i,j} = 1$ if $B_{i,j} \neq 0$ and $L_{i,j} = 0$ otherwise. The bipartite graph $\mathcal{G}(L)$ has the edge set

$$E = \{\{1,7\},\{1,8\},\{1,9\},\{2,10\},\{2,11\},\{3,10\},\{3,12\},\{4,7\},\{4,10\},\{5,11\},\{6,12\}\},$$

see Fig. 1(c). In the second step, the maximum matching

$$M = \{\{1,8\},\{2,10\},\{4,7\},\{5,11\},\{6,12\}\}$$

in $\mathcal{G}(L)$ is found. In the third step, the Hungarian forest shown in Fig. 1(d) is found. The set $\{2,3,5,6\}$ of vertices is a fundamental circuit of the row matroid of $\mathcal{G}(L)$. Its elements correspond to Kirchhoff's current equations of the nodes 2 and 3 of the network from Fig. 1(a) and the element equations of C_1 and C_2. Indeed, these equations are linearly dependent. This corresponds to the fact that the branches C_1, C_2 form a cut-set of the network graph.

In the fourth step, we find the fundamental circuit $\{8, 9\}$ in the column matroid of $\mathcal{G}(L)$. Its elements correspond to the voltages v_{C_1} and v_{C_2}. Indeed, the corresponding columns of B are linearly dependent. The voltages v_{C_1} and v_{C_2} cannot be determined uniquely due to the cut-set composed of C_1 and C_2.

3.2 Example The electrical network from Fig. 2(a) is a comparator which is commonly used for applications in power amplifiers and motor control, see [11], pp. 6-87. In a very first attempt to simulate that network, it seems natural to regard the OpAmps as ideal, i.e., to model them as nullors. The network equations of the resulting network from Fig. 2(b) are of the form (3) with $x = (v_1, \ldots, v_8, i_i, \ldots, i_8)$, $A = 0$, and B equal to the matrix

$$\begin{pmatrix}
-1 & 1 & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & -1 & 1 & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & 1 & 1 & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & 1 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & 1 & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & -1 & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & -1 & 1 & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & R & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & R' & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & R'' & \cdot
\end{pmatrix}.$$

First, define L by $L_{i,j} = 1$ if $B_{i,j} \neq 0$ and $L_{i,j} = 0$ otherwise. In the second step, a maximum matching of cardinality 14 is found. In the third and fourth steps, the two circuits $\{1, 6, 7, 9, 10, 11, 12, 15\}$ and $\{2, 6, 11, 13, 14, 15\}$ of the row matroid and the two circuits $\{6, 8\}$ and $\{7, 14, 15, 16\}$ of the column matroid of $\mathcal{G}(L)$ are found. The corresponding rows and columns of B are linearly dependent. While the circuits of the row matroid are difficult to interpret, those of the column matroid correspond to the singular subnetwork composed of the branches 6, 7 and 8.

4. CONCLUSIONS

We have proposed a method for detecting inconsistencies in electrical networks based on the calculation of fundamental circuits of two matroids induced by a bipartite graph derived from the network equations.

That method has been implemented in the circuit simulator TITAN of Infineon Technologies and, despite its simplicity, is of great help in locating inconsistencies in erroneous network descriptions, especially if these descriptions contain VHDL-AMS code.

In addition, our method is extremely fast, which allows for its routine application prior to any simulation: In tests on network equations without inconsistencies involving up to 120000 variables, the authors have never observed running times exceeding 1 second on standard workstations.

We would like to emphasize that fundamental circuits of the kind we determine in our method have been employed for other purposes earlier and that there are methods different from ours to calculate them [12–14].

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5. APPENDIX

We give basic definitions and well-known results from [15].

5.1. Basic Terminology

A *graph* is a pair (V, E) of a finite set V of *vertices* and a set of *edges* with $E \subseteq \{\{v_1, v_2\} | v_1, v_2 \in V\}$. Let G be a graph, $G = (V, E)$, and $M \subseteq E$. The set of end vertices of the edges in M is denoted by ∂M. M is a *matching in G* if different edges of M do not have an end vertex in common. A matching M in G is *perfect* if $\partial M = V$. A set $C \subseteq V$ is a *vertex cover in G* if $e \cap C \neq \emptyset$ for all $e \in E$.

5.2. Bipartite graphs, Hungarian forests

Let G be a graph, $G = (V, E)$, and M a matching in G. G is *bipartite* if there is a partition $\{V_1, V_2\}$ of V such that $E \subseteq \{\{v_1, v_2\} | v_1 \in V_1, v_2 \in V_2\}$. In that case, we also call (V_1, V_2, E) a graph, and the following terms are meant to be defined with respect to the partition $\{V_1, V_2\}$ and with respect to the order in which V_1 and V_2 appear in (V_1, V_2, E).

Let G be a bipartite graph, $G = (V_1, V_2, E)$, M a matching in G, and F a forest in G, $F = (V_1', V_2', E')$, $V_1' \subseteq V_1$, $V_2' \subseteq V_2$. F is *M-alternating* if

(i) for all $v \in V_2'$, the degree of v in F is 2 and $v \in \partial(M \cap E')$,

(ii) each component of F has a vertex contained in $V_1 \setminus \partial M$.

F is a *Hungarian* forest in G if F is a maximal M-alternating forest in G and $\{\{v_1, v_2\} \in E | v_1 \in V_1', v_2 \in V_2 \setminus \partial M\} = \emptyset$ for some matching M in G.

5.1 Theorem *Under the assumptions and in the notation of Theorem 2.1, M is maximum iff F is Hungarian. Moreover, if M is maximum, then $(V_1 \setminus V_1') \cup V_2'$ is a minimum vertex cover in G.*

5.3. Matroids, bipartite graphs, matrices

Let M be a matroid, $M = (S, F)$. A *circuit in M* is a minimal dependent set. Let G be a bipartite graph, $G = (V_1, V_2, E)$, and $F = \{V_2 \cap \partial M | M \text{ is a matching in } G.\}$. Then (V_2, F) is a matroid which is called the *column matroid of G* and denoted $\mathcal{C}(G)$. The *row matroid* $\mathcal{R}(G)$ is a matroid with ground set V_1 defined analogously.

Let $L : \mathbb{R}^k \to \mathcal{L}(\mathbb{K}^m, \mathbb{K}^n)$ be an $n \times m$ matrix over $\mathbb{K}$, $\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$, that depends analytically on some parameter $p \in \mathbb{R}^k$. Let $S = \{n + 1, \ldots, n + m\}$ and

$$F = \{n + X | X \subseteq \{1, \ldots, m\}, \exists_{p \in \mathbb{R}^k} \text{The set of columns of } L(p) \text{ corresponding to } X \text{ is linearly independent.}\}.$$

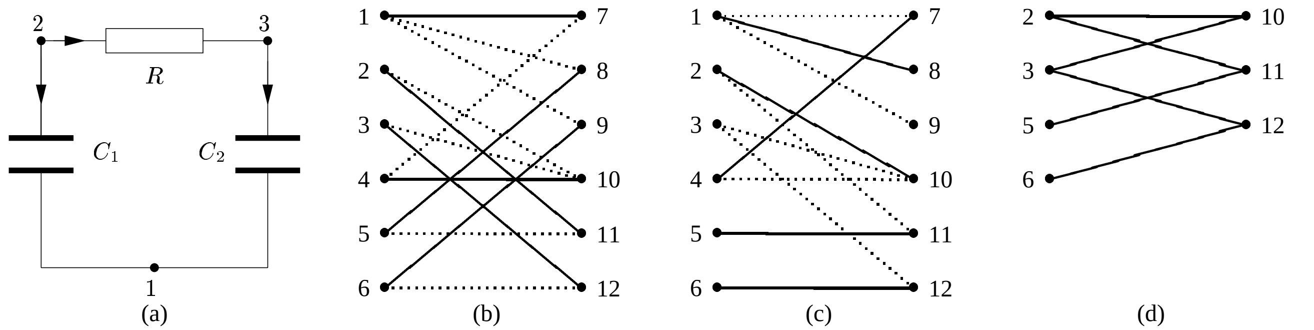

Fig. 1. Illustration of example 3.1. (a) Electrical network. (b) Bipartite graph $\mathcal{G}(L)$. (c) Bipartite graph $\mathcal{G}(L)$. (d) Hungarian forest in $\mathcal{G}(L)$. (In (b) and (c), matching edges are solid lines, while all other edges of the graphs are shown as dotted lines.)

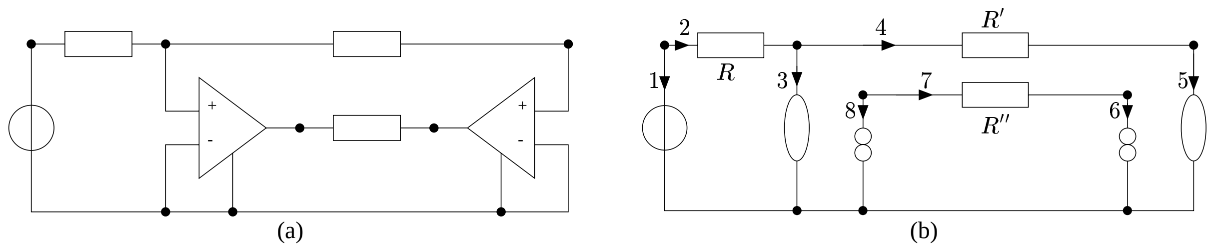

Fig. 2. Illustration of example 3.2. (a) Electrical network. (b) OpAmps modeled by nullors.

Then (S, F) is a matroid which is called the *column matroid of* L and denoted $\mathcal{C}(L)$. The *row matroid* $\mathcal{R}(L)$ is a matroid with ground set $\{1, \ldots, n\}$ defined analogously.

Denote by $\operatorname{supp} L$ the set of positions of the nonzeros of L, $\operatorname{supp} L = \{(i,j) | 1 \leq i \leq n, 1 \leq j \leq m, \exists_{p \in \mathbb{R}^k} L(p)_{i,j} \neq 0\}$. The *bipartite graph of* L, $\mathcal{G}(L)$, is the bipartite graph (V_1, V_2, E) with $V_1 = \{1, \ldots, n\}$, $V_2 = \{n+1, \ldots, n+m\}$, and $E = (0, n) + \operatorname{supp} L$, where '$+$' denotes elementwise addition. The matrix L is *structurally regular* if its bipartite graph $\mathcal{G}(L)$ has a perfect matching.

6. REFERENCES

[1] U. Feldmann, U. A. Wever, Q. Zheng, R. Schultz, and H. Wriedt, "Algorithms for modern circuit simulation", *AEÜ*, vol. 46, no. 4, pp. 274–285, 1992.

[2] Schultz, *Programm-Dokumentation TITAN-02 – Topologie-Test und Row-Swapping – Programm-Modul TOPCHK*, SIEMENS AG - Muenchen – ZTI DES 251, July 1988.

[3] G. Reißig, "Extension of the normal tree method", *Internat. J. Circuit Theory Appl.*, vol. 27, no. 2, pp. 241–265, 1999, Erratum in vol. 28, no. 1, 2000, p. 99.

[4] K. Röbenack and G. Reißig, "Successive index reduction of electrical networks with a minimum number of additional network elements", in *Proc. 43rd Intl. Sci. Colloqu., Techn. Univ. Ilmenau, Sept. 21–24, 1998*, W. Gens, Ed., vol. 3, pp. 192–197, (in German).

[5] W. Morton and C. Collingwood, "An equation analyser for process models", *Computers chem. Engng.*, vol. 22, no. 4/5, pp. 571–585, 1998.

[6] J. A. Paloschi, "An algorithm for efficient specification analysis in large-scale dynamic process simulation", in *Europ. Symp. Computer Aided Process Eng. - 10*, S. Pierucci, Ed. 2000, pp. 229–234, Elsevier.

[7] K. Murota, *Systems Analysis by Graphs and Matroids: Structural Solvability and Controllability*, Springer, 1987.

[8] IEEE, *Definition of Analog and Mixed Signal Extensions to IEEE Standard VHDL, IEEE Standard 1076.1-1999*.

[9] E. Christen, K. Bakalar, A. M. Dewey, and E. Moser, "Analog and mixed-signal modeling using the VHDL-AMS language", in *36th Design Automation Conference (DAC), New Orleans*, 21-25 June 1999.

[10] R. K. Ahuja, T. L. Magnanti, and J. B. Orlin, *Network flows*, Prentice Hall Inc., 1993.

[11] Texas Instruments, *Operational Amplifiers and Comparators, Data Book, vol. B*, 1995.

[12] C. C. Pantelides, "The consistent initialization of differential-algebraic systems", *SIAM J. Sci. Statist. Comput.*, vol. 9, no. 2, pp. 213–231, Mar. 1988.

[13] J. R. Gilbert, "Predicting structures in sparse matrix computations", *SIAM J. Matrix Anal. Appl.*, vol. 15, no. 1, pp. 62–79, 1994.

[14] D. M. Collins and P. I. Barton, "Efficient construction of linear state-space models from index one DAEs", Tech. Rep., MIT, Dept. Chem. Eng., 77 Massachusetts Av., Cambridge, MA 02139, U.S.A., 16 Aug. 2001.

[15] L. Lovàsz and M. D. Plummer, *Matching Theory*, 1986.