Index one regularization of networks using a minimum number of additional elements

Klaus Röbenack and Gunther Reißig

Technische Universität Dresden, Sonderforschungsbereich 358, Teilprojekt D5 (Prof. K. J. Reinschke)
Mommsenstr. 13, 01062 Dresden, Germany

Abstract. Many numerical problems in circuit simulation are caused by the fact that the underlying differential algebraic equations (DAE) are singular or have an index greater than one. A new method for an index one regularization of possibly singular networks is presented here. It can be applied to linear electrical networks containing resistors, capacitors, inductors, independend voltage and current sources, and nullator-norator pairs. For a subclass of networks, including all regular ones, our method uses the minimum number of additional elements possible.

1 Introduction

Many physical systems — especially electrical networks — can be modelled most naturally by *differential algebraic equations (DAE)*. The *regularity* as well as the *index* of a DAE play crucial roles concerning analytical and numerical properties [1–3]. For example on singular and higher index problems small perturbations of the input may cause arbitrarly large errors in the solution [3].

One approach to overcome the disadvantages related to singular or higher index problems in circuit simulation is the modification of the electrical network. In this paper, we consider the following modifications to be admissible:

(i) Augmenting the network with either a resistor, a capacitor or an inductor in series to an existing branch.
(ii) Augmenting the network with either a resistor, a capacitor or an inductor as a new branch between two existing nodes.

It is well-known that index one networks can be generated from regular networks using these modifications only, see [4] and references cited there. The method proposed in this paper works for all regular networks and all singular networks that can be made regular by the modifications above. Furthermore, our method generates regular networks of index one using the minimum number of additional elements possible.

For singular networks that cannot be made regular by the modifications (i) and (ii) alone, we propose an extension of our regularization method which again yields regular networks of index one. Unfortunately, we cannot claim minimality in that case.

2 Preliminaries

We confine ourselves to linear networks containing the following types of elements: independent voltage and current sources, time–invariant resistors, capacitors, inductors, and nullator–norator pairs. Furthermore, we assume the network graph to be connected.

Consider such a network $\mathcal{N}$ consisting of b branches. Let $p \in \mathbb{R}^k$ denotes the vector containing all parameters of $\mathcal{N}$, i.e., resistances, capacitances, and inductances. The *circuit equations* of $\mathcal{N}$ are given by

$$A(p)\,\dot{x} + B(p)\,x = q(t),$$

where A and B are parameter dependent $2b \times 2b$-matrices, x denotes the $2b$-dimensional vector of the branch voltages and the branch currents, and q denotes the vector of the independend sources.

A network $\mathcal{N}$ is said to be *regular* or *solvable* if $s \mapsto \det(sA(p) + B(p))$ is not the zero polynomial [5]. Otherwise, the network $\mathcal{N}$ is called *singular*. The degree of $s \mapsto \det(sA(p) + B(p))$ with respect to s is called the *dynamic complexity*. A network $\mathcal{N}$ is said to be of *index* ν if the regular part of the associated matrix pencil $(A(p), B(p))$ has the index ν, i.e., the index is the size of the greatest Jordan block associated with a characteristic root at infinity, see [6, 1, 7]. A network property is said to be met *generically* if the property under consideration is met for all p belonging to an open and dense subset of $\mathbb{R}^k$.

3 The Algorithm

In order to formulate our algorithm we need to introduce the concept of pairs of conjugate trees, see [8,9].

Definition 1. *Let G be the undirected network graph of some network $\mathcal{N}$. A pair (t_1, t_2) of spanning trees of G is called a* pair of conjugate trees (PCT) *if*

(1) t_1 contains all norator branches, no nullator branch, all voltage source branches, no current source branch, and some resistor, capacitor or inductor branches, and
(2) t_2 contains all nullator branches, no norator branch, all voltage source branches, no current source branch, and the same resistor, capacitor or inductor branches as t_1.

Let C and L denote the sets of capacitor and inductor branches of $\mathcal{N}$, respectively, and let us denote the cardinality of any set H by $|H|$.

Definition 2. *A pair of conjugate trees (t_1, t_2) is called* normal [4] *if it maximizes the sum of the number of tree capacitors and the number of co-tree inductors among all pairs of conjugate trees of the network $\mathcal{N}$ in question, i.e., if*

$$\max\{|\tau_1 \cap C| + |L \setminus \tau_1| : (\tau_1, \tau_2) \textit{ is a PCT of } \mathcal{N}\}$$

equals $|t_1 \cap C| + |L \setminus t_1|$.

The algorithm is given in Tab. 1. The network $\mathcal{N}_2$ obtained from this algorithm is generically of index one. The correctness of the algorithm is implied by the follwing theorem:

Theorem 1. *Let $\mathcal{N}$ be a linear network containing the following types of elements: independent voltage and current sources, time–invariant resistors, capacitors, inductors, and nullator–norator pairs. We assume the network graph to be connected and that there exists a generically regular network resulting form $\mathcal{N}$ applying the modifications (i) and (ii). Then the network $\mathcal{N}_2$ obtained from the algorithm is generically regular and of index one. Furthermore, the network $\mathcal{N}_2$ minimizes the number of additional elements among all generically index one networks obtained from $\mathcal{N}$ using admissible modifications only.*

Input:	Network $\mathcal{N}$.
Step 1:	Supplement $\mathcal{N}$ by additional inductor branches between each pair of different nodes, which are not yet connected by one branch. The voltage sources have to be replaced by capacitors and the current sources have to be replaced by inductors (see Table 2). We denote the resulting network by $\mathcal{N}_1$.
Step 2:	Determine a normal pair of conjugate trees (t_1, t_2) of $\mathcal{N}_1$.
Step 3:	Construct a network $\mathcal{N}_2$ that is obtained from $\mathcal{N}$ as follows: (1) Augment $\mathcal{N}$ with either a resistor or an inductor in series to each capacitor branch or voltage source branch of $\mathcal{N}$ not contained in t_1. (2) Augment $\mathcal{N}$ with either a resistor or a capacitor in parallel to each inductor branch of current source branch of $\mathcal{N}$ contained in t_1. (3) Augment $\mathcal{N}$ with either a resistor or a capacitor between each pair of nodes connected by an inductor branch that is contained in t_1 and $\mathcal{N}_1$ but not in $\mathcal{N}$.
Output:	Network $\mathcal{N}_2$.

Table 1. The algorithm

Before we give the proof we would like to remind the reader of a network-theoretic result. Let $\mathcal{N}$ be a network as mentioned above. Then there holds [4, Cor. 4.9]:

(1) $\mathcal{N}$ is generically regular if and only if $\mathcal{N}$ has a pair of conjugate trees.
(2) $\mathcal{N}$ is generically of index one if and only if $\mathcal{N}$ has a pair of conjugate trees (t_1, t_2) with $C \subseteq t_1$ and $L \cap t_1 = \emptyset$.

Now, we are able to prove Theorem 1.

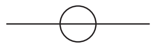
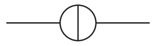
$\mathcal{N}$	$\mathcal{N}_1$
	
	

Table 2. Replacements in step 1

Theorem 2. *Let us denote the resistor, inductor and capacitor branches of $\mathcal{N}$ by R, L and C, respectively. To simplify further notations, we will denote the voltage source branches of $\mathcal{N}$ as well as the capacitor branches of $\mathcal{N}_1$ resulting from the replacements according to Table 2 by C_{VS}, and the current source branches of $\mathcal{N}$ as well as the inductor branches of $\mathcal{N}_1$ resulting from this replacements by L_{CS}. Moreover, the additional inductor branches inserted in step 1 will be denoted by L_A. We assumed that there exists a generically regular network resulting form $\mathcal{N}$ applying the replacements shown in Table 2. This implies that the network $\mathcal{N}_1$ has a normal pair of conjugate trees (t_1, t_2).*

Part I: *Now, we have to prove that $\mathcal{N}_2$ is generically of index one. For this purpose we consider different modifications of the network $\mathcal{N}_1$:*

(a) Augmenting the network with an additional branch in series to each $c \in C \setminus t_1$ yields a new network with a PCT $(t_1 \cup C, t_2 \cup C)$.
(b) Augmenting the network with an additional branch in series to each $c \in C_{VS} \setminus t_1$ yields a new network with a PCT $(t_1 \cup C \cup C_{VS}, t_2 \cup C \cup C_{VS})$.
(c) By augmenting the resulting network with a branch in parallel to each $l \in L \cap t_1$ and denoting the set of supplemented branches by Z, we obtain a network with a PCT $((t_1 \cup C \cup C_{VS} \cup Z) \setminus L, (t_2 \cup C \cup C_{VS} \cup Z) \setminus L))$.
(d) By augmenting the resulting network with a branch in parallel to each $l \in L_{CS} \cap t_1$ and denoting the set of supplemented branches by Y, we obtain a network with a PCT $((t_1 \cup C \cup C_{VS} \cup Z \cup Y) \setminus (L \cup L_{CS}), (t_2 \cup C \cup C_{VS} \cup Z \cup Y) \setminus (L \cup L_{CS}))$.
(e) Replacing each $l \in t_1 \cap L_A$ by either a resistor or a capacitor yields a new network with the same PCT as before.
(f) Deleting each $l \in L_A \setminus t_1$ yields a modified network with the same PCT as above.

Now, if we choose the new branches in (a)-(d) according to step 3(i),(ii) of our algorithm, the network obtained here is nothing else than $\mathcal{N}_2$. The construction above yields a normal pair of conjugate trees which contains no inductor branch and all capacitor branches of the network. Hence, the network $\mathcal{N}_2$ is generically of index one [4, Cor. 4.9].

Part II: *It remains to show that $\mathcal{N}_2$ minimizes the number of additional network elements. In this part we consider the generic complexity σ of $\mathcal{N}_1$. Let b and b_2 denote the number of branches on $\mathcal{N}$ and $\mathcal{N}_2$, respectively, and let $\Delta b_2 := b_2 - b$. Then there holds (cf. [4, Th. 4.5])*

$$\begin{aligned}\sigma &= |(C \cup C_{VS}) \cap t_1| + |(L \cup L_{CS} \cup L_A) \setminus t_1| \\ &= |C| + |C_{VS}| + |L| + |L_{CS}| + |L_A| - \underbrace{|C \setminus t_1|}_{(a)^1} - \underbrace{|C_{VS} \setminus t_1|}_{(b)^1} - \underbrace{|L \cap t_1|}_{(c)^1} - \underbrace{|L_{CS} \cap t_1|}_{(d)^1} - \underbrace{|L_A \cap t_1|}_{(e)^1} \\ &= |C| + |C_{VS}| + |L| + |L_{CS}| + |L_A| - \Delta b_2.\end{aligned} \tag{1}$$

In order to show that $\mathcal{N}_2$ contains a minimal number of additional network elements we consider a network $\tilde{\mathcal{N}}$ consisting of $\tilde{b}$ branches and obtained from $\mathcal{N}$ using the admissible modifications with resistor branches only (the other replacements can be treated similarly). Assume $\tilde{\mathcal{N}}$ to be generically regular, of index one, and to be minimal with respect to the additional network elements. Then, it can be verified that $\tilde{\mathcal{N}}$ may have been obtained from $\mathcal{N}$ using the following modifications only:

(1) Augmenting $\mathcal{N}$ with a resistor in series to a capacitor.
(2) Augmenting $\mathcal{N}$ with a resistor in series to a voltage source.
(3) Augmenting $\mathcal{N}$ with a resistor in parallel to an inductor.

[1] Resulting from additional elements according to the first part of the proof.

(4) Augmenting $\mathcal{N}$ with a resistor in parallel to an current source.
(5) Augmenting $\mathcal{N}$ with a resistor as a new branch between two nodes which are not yet connected by one branch of $\mathcal{N}$.

We denote the sets of additional resistor branches resulting from (1)-(5) by R_C, R_{VS}, R_L, R_{CS} and R_A, respectively. Let $(\tilde{t}_1, \tilde{t}_2)$ denote a normal pair of conjugate trees of $\tilde{\mathcal{N}}$ and $\Delta\tilde{b} := \tilde{b} - b := |R_C| + |R_{VS}| + |R_L| + |R_{CS}| + |R_A|$ the number of additional branches. The generic regularity implies $C_{VS} \subseteq \tilde{t}_1$ and $L_{CS} \cap \tilde{t}_1 = \emptyset$. Because we assumed the generic index to be one we have $C \subseteq \tilde{t}_1$ and $L \cap \tilde{t}_1 = \emptyset$. Hence, the generic complexity of $\tilde{N}$ is given by $|C| + |L|$. Now, we modify $\tilde{N}$ as follows:

(a) Replace each voltage source by a capacitor and each current source by an inductor (Table 2). The generic complexity of the resulting network is $|C| + |C_{VS}| + |L| + |L_{CS}|$.
(b) Contraction of each $r \in R_C$. The new network has a generic complexity not less than $|C| + |C_{VS}| + |L| + |L_{CS}| - |R_C|$.
(c) Contraction of each $r \in R_{VS}$. The new network has a generic complexity not less than $|C| + |C_{VS}| + |L| + |L_{CS}| - |R_C| - |R_{VS}|$.
(d) Delete each $r \in R_L$. The generic complexity will be not less than $|C| + |C_{VS}| + |L| + |L_{CS}| - |R_C| - |R_{VS}| - |R_L|$.
(e) Delete each $r \in R_{CS}$. The generic complexity will be not less than $|C| + |C_{VS}| + |L| + |L_{CS}| - |R_C| - |R_{VS}| - |R_L| - |R_{CS}|$.
(f) Replace each $r \in R_A$ by an inductor. The lower bound of the generic complexity remains the same as before.
(g) Supplement the network by additional inductor branches between each pair of nodes not connected by one branch in $\tilde{\mathcal{N}}$. The generic complexity $\tilde{\sigma}$ of the resulting network fulfills

$$\begin{aligned} \tilde{\sigma} &\geq |C| + |C_{VS}| + |L| + |L_{CS}| + |L_A| - |R_C| - |R_{VS}| - |R_L| - |R_{CS}| - |R_A| \\ &= |C| + |C_{VS}| + |L| + |L_{CS}| + |L_A| - \Delta\tilde{b}. \end{aligned} \tag{2}$$

The network obtained here is nothing else than $\mathcal{N}_1$, i.e., $\tilde{\sigma} = \sigma$. Because of Eqs. (1) and (2) we have $\Delta b_2 \leq \Delta\tilde{b}$, i.e., the network $\mathcal{N}_2$ has a minimum number of additional elements.

Step 1 and 3 of the algorithm require at most $\mathcal{O}(b^2)$ simple network modifications. The normal pair of conjugate trees (see step 2) can be determined using the method presented in [10]. The complexity of the method presented there is $\mathcal{O}(b^4)$. This implies that the number of operations of the whole algorithm is bounded by a polynomial in b. Because the network $\mathcal{N}_1$ (step 1) may contain $\mathcal{O}(b^2)$ branches, the complexity of the whole algorithm is $\mathcal{O}(b^8)$.

Corollary 1 (To the proof of Theorem 1). *Let $\mathcal{N}$, C, C_{VS}, L, L_{CS}, L_A and σ be defined as above. Then the minimal number of additional network elements required to establish an index on regularization of $\mathcal{N}$ using the admissible modifications is equal to $|C| + |C_{VS}| + |L| + |L_{CS}| + |L_A| - \sigma$.*

4 Regularization of general singular Networks

In this section we will propose an extension of our algorithm which yields a generially regular index one network. The extended algorithm is given in Table 3.

Input:	Network $\mathcal{N}$.
Step 0:	Augment $\mathcal{N}$ with two capacitors in series to each nullator and norator (see Table 4). We denote the resulting network by $\mathcal{N}'$.
Step 1–2:	Apply the algorithm shown in Table 1 to the network $\mathcal{N}'$.
Output:	Network $\mathcal{N}_2$.

Table 3. The extended algorithm

The following theorem implies the correctness of this regularization. Observe that we do not claim minimality for general singular networks.

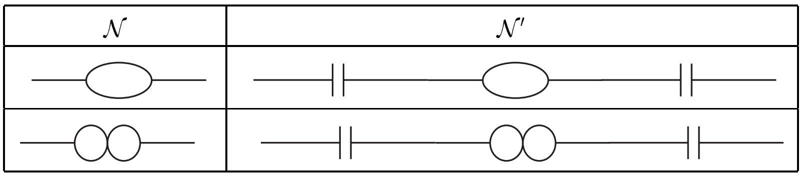


Table 4. Replacements of network elements

Theorem 3. *Let $\mathcal{N}$ be a linear network containing the following types of elements: independent voltage and current sources, time–invariant resistors, capacitors, inductors, and nullator–norator pairs. We assume the network graph to be connected. Then the network $\mathcal{N}_2$ obtained from the extended algorithm is generically regular and of index one.*

To prove Theorem 2 we need the following graph-theoretic statement:

Let $G = (V, E)$ be an undirected complete graph (i.e., all pairs of nodes are connected by a branch) with the set V of nodes and the set E of branches. Let be $E_1, E_2 \subset E$, $E_1 \cap E_2 = \emptyset$ and $|E_1| = |E_2|$. Furthermore, assume E_1 and E_2 to be circuit free. Then there exists a set $E_3 \subset E \setminus (E_1 \cup E_2)$ such that $T_1 := E_1 \cup E_3$ as well as $T_2 := E_2 \cup E_3$ are spanning trees.

Theorem 4. *In the following we denote the set $\tilde{V}$ of nodes that are incident to a set $\tilde{E}$ of branches by $\tilde{V} = V(\tilde{E})$. First, we will show that there exists a set $E_3 \subset E \setminus (E_1 \cup E_2)$ such that $T_1 := E_1 \cup E_3$ and $T_2 := E_2 \cup E_3$ are circuit free and cover all nodes. There is at least one set $E_3 := \emptyset$ such that T_1 and T_2 are circuit free (according to our assumption). Now, let E_3 be chosen in such a way that it maximizes the number $|V(T_1)| + |V(T_2)|$ of all nodes covered by T_1 and T_2. Assume $|V(T_1)| < |V(E)|$, i.e., T_1 does not cover all nodes (contrary to assertion). Then there exists a non empty set $\{v_1, \ldots, v_s\} = V(E \setminus T_1)$ and a (only possibly non empty) set $\{w_1, \ldots, w_t\} = V(E \setminus T_2)$ not covered by T_1 and T_2, respectively. We have to consider three different cases:*

(1) There are two equal nodes $v_j = w_k$. First, we assume $V(T_1) \cap V(T_2) = \emptyset$. This implies $E_3 = \emptyset$. The sets $V_1 := V(E_1)$ and $V_2 := V(E_2)$ are disjoint and non empty. Hence, there are nodes $u_{i_1} \in V_1 \setminus V_2$ and $u_{i_2} \in V_2 \setminus V_1$. We will denote a branch between the nodes u_{i_1} and u_{i_2} by $u_{i_1}u_{i_2}$. The new set $E_3' := \{u_{i_1}u_{i_2}\}$ yields a pair $T_1' := E_1 \cup E_3'$ and $T_2' := E_2 \cup E_3'$ of circuit free sets of edges with $|T_i'| = |T_i| + 1 > |T_i|$ $(i = 1, 2)$, see Fig. 1(a). This contradicts our assumption with respect to the maximiality of $|T_1| + |T_2|$.
Now, let $V(T_1) \cap V(T_2) \neq \emptyset$, i.e., there exists a node $u \in V(T_1) \cap V(T_2)$. Clearly, $u \neq v_j$. Using $E_3' := E_3 \cup \{uv_j\}$ one obtains two associated circuit free sets T_1', T_2' with $|T_i'| > |T_i|$ (Fig. 1(b)). Contradiction.

(2) There are no equal nodes v_j and w_k. In this case, the branch v_jw_k is isolated with respect to T_1 and T_2. Defining $E_3' := E_3 \cup \{v_jw_k\}$ one gets two circuit free sets T_1' and T_2' with a greater cardinality than T_1 and T_2, cf. Fig. 1(c). This contradicts our assumption.

(3) Assume T_2 covers all nodes, but not T_1, i.e., $V = V(T_2) = V(T_1) \cup \{v_1, \ldots, v_j, \ldots, v_s\}$. Because of $|T_1| = |T_2|$ and $|V(T_1)| < |V(T_2)|$ the circuit free set T_2 consists of at least two different components C_1 and C_2. Clearly, each component is a tree. Without loss of generality we assume $v_j \in V(C_1)$. Choose an arbitrary $u \in V(C_2)$, which implies $v_j \neq u$. The branch v_ju connects the trees C_1 and C_2. With $E_3' := E_3 \cup \{v_ju\}$ one obtains a new circuit free set T_2'. We have to analyse whether the associated set T_1' is circuit free or not. In case of $u \in V(T_1)$ a tree of T_1 will be extended by a single leaf, i.e., $T_1' := T_1 \cup \{v_ju\}$ remains to be a circuit free. Otherwise, both nodes $u, v_j \notin V(T_1)$ are isolated with respect to T_1. Obviously, T_1' is circuit free, too. Our construction of T_1', T_2' contradicts the maximality of $|T_1| + |T_2|$.

Now, we have proved that there is a set $E_3 \subset E \setminus (E_1 \cup E_2)$ such that T_1 and T_2 are circuit free and cover all nodes. Because of $|T_1| = |T_2|$ and $|V(T_1)| = |V(T_2)| (= |V|)$ the sets T_1 and T_2 consists of the same number l of trees. We will denote the trees of T_1 and T_2 by $C_1, \ldots, C_l$ and $D_1, \ldots, D_l$, respectively. If $l = 1$ each circuit free set T_1, T_2 is a tree itself. Let $l > 1$. Then there are integers $j_1, j_2, k_1, k_2 \in \{1, \ldots, l\}$ with $j_1 \neq j_2$ and $k_1 \neq k_2$ such that $C_{j_1} \cap D_{k_1} \neq \emptyset$ and $C_{j_2} \cap D_{k_2} \neq \emptyset$. Choose $v \in C_{j_1} \cap D_{k_1}$ and $w \in C_{j_2} \cap D_{k_2}$. In both cases of T_1 and T_2 the branch vw connects two trees. Hence, using $E_3' := E_3 \cup \{vw\}$ the resulting *trees T_1' and T_2' are circuit free with exactly $l-1$ trees. This procedure can be applied until one obtains a pair of spanning trees.*

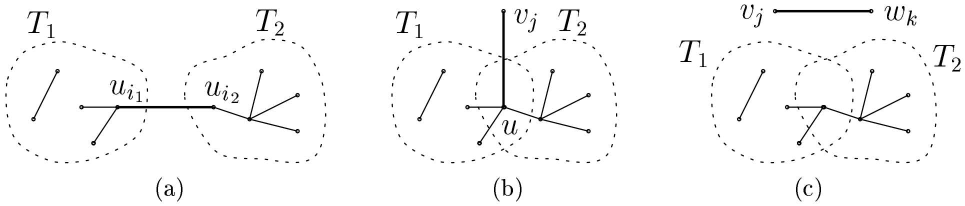


Fig. 1. Sketch to the proof of Lemma 4

Proof (of Theorem 2). First, we will show that $\mathcal{N}_1$ has a pair of conjugate trees. After performing step 0 and step 1 of the extended algorithm one obtains a network $\mathcal{N}_1$ containing at least one branch between all pairs of node, i.e., the network graph G is complete. Let be E_1 and E_2 the sets of norator and nullator branches, respectively. Obviously, E_1 and E_2 are disjoint. The replacements according to step 0 imply that E_1 and E_2 are circuit free. (More precisely, E_1 and E_2 consists of isolated branches.) Because we assumed $\mathcal{N}$ containing nullator-norator pairs but not single nullators or norators the sets E_1 and E_2 have the same cardinality. It follows from Lemma 4 that there exists a set E_3 of branches such that $t_1 := E_1 \cup E_3$ and $t_2 := E_2 \cup E_3$ are spanning trees. All branches of $\mathcal{N}_1$ which are neither a norator nor a nullator branch can be resistor, inductor or capacitor branches only. Hence, t_1 contains all norator branches, no nullator branch, and some resistor, inductor or capacitor branches, and t_2 contains all nullator branches, no norator branch, and the same resistor, inductor or capacitor branches as t_1. In other words, (t_1, t_2) is a pair of conjugate trees of $\mathcal{N}_1$.

Now, we have proved that the network $\mathcal{N}_1$ obtained from the extended algorithm is generically regular. Applying Part I of the proof of Theorem 1 shows that the associated network $\mathcal{N}_2$ is generically regular and of index one. ■

5 Application of the Algorithm

Example 1. Consider the network $\mathcal{N}$ given by Fig. 2(a). This network is generically of index four. It is the linear counterpart of a so–called *inverse system* used to decode chaotically modulated signals [11]. Augmenting $\mathcal{N}$ with two capacitors in series to the nullator and the norator (according to Table 4) yields the network $\mathcal{N}'$. To obtain the associated network $\mathcal{N}_1$ we have to replace the current source by one inductor, and have to augment the resulting network by further inductors. Using a normal pair of conjugate trees of $\mathcal{N}_1$ (see Fig. 2(b),(c)) and applying step 4 of the algorithm one gets a generically regular index one network $\mathcal{N}_2$ (Fig. 2(d),(e)). Note that the procedure presented here requires only two instead of three additional elements.

Example 2. The circuit shown in Fig. 3(a) is a simple differential comparator. The output load resistor R is controlled by a bridge circuit realized by two operational amplifiers. This circuit could be part of a switching power amplifier [12, p. 6-17] or a pulse-width-modulated motor speed controller (e.g., [12, p. 6-87]). The associated linearized and idealized network $\mathcal{N}$ is given in Fig. 3(b), where we use nullator-norator pairs instead of operational amplifiers. Because the nullators are in parallel, the network $\mathcal{N}$ cannot contain a pair of conjugate trees. It is possible to regularize $\mathcal{N}$ using our algorithm. Augmenting $\mathcal{N}$ with a single resistor R' (see Fig. 3) one obtains the associated network $\mathcal{N}_2$. Observe that such a resistor R' between the inputs of real operational amplifiers (among other resistors connected with ground and power) is very common to achieve different input triggering voltages, e.g. window comparators [13, pp. 128–130] and two-phase nonoverlapping clock generators [12, p. 6-88].

Example 3. The last example deals with a network consisting of two components, i.e., the network graph is not connected. A possible application is a digital transmission isolator realized by an optical coupling device (optical coupler). Optical couplers may be modelled by controlled sources. Such a network has been sketched in Fig. 4(a). It contains a voltage-controlled current source. (Note that in practical applications

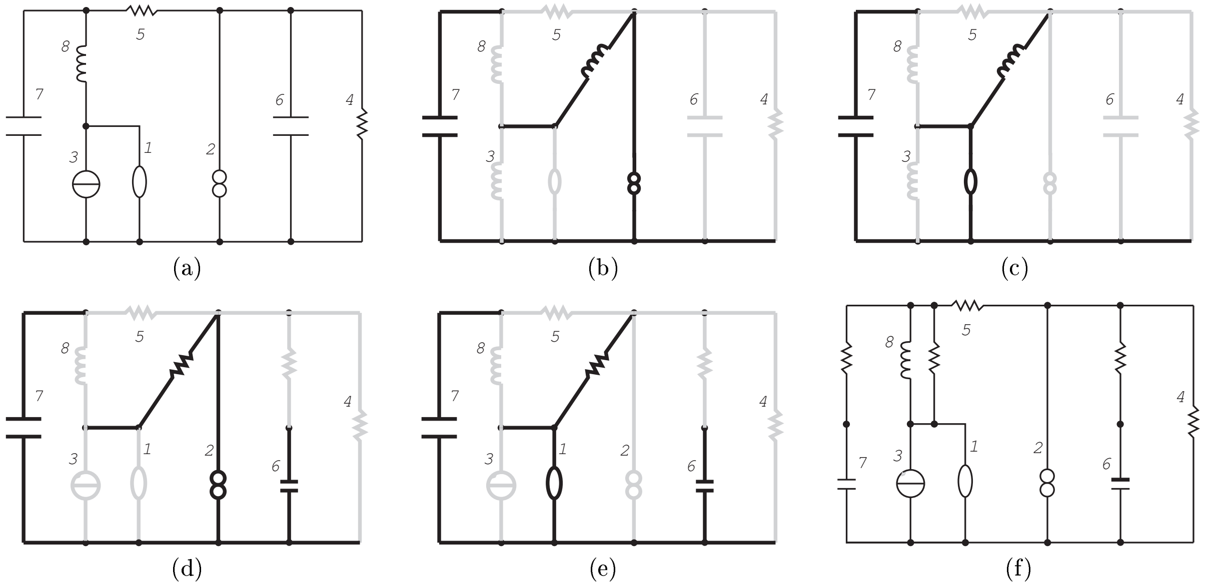


Fig. 2. (a) Network $\mathcal{N}$, (b),(c) Network $\mathcal{N}_1$ with a normal pair of conjugate trees (t_1, t_2), where all inductor brnanches of L_A not contained in (t_1, t_2) have been omitted, (d),(e) Resulting network $\mathcal{N}_2$ with a normal pair of conjugate trees, (f) Network modified as in [4] without optimization

voltage-controlled voltage sources using light-to-voltage converters as optical receiver circuits are more common, but all four types of controlled sources may cause the singularity described here.) The network is generically regular and of index two. A replacement of the controlled source by an *"equivalent"* network composed of nullators, norators and a linear resistor (see [9, pp. 24-25]) yields a generically singular network $\mathcal{N}$ (Fig. 4(b)) with index two. Applying our algorithm one gets a generically regular index two network $\mathcal{N}_2$ using two additional elements, see Fig. 4(c). Hence, it is impossible to augment $\mathcal{N}$ by one additional branch such that the resulting network is generically regular *and* of index one at the same time: If we supplement $\mathcal{N}$ with a branch between one pair of nodes $(1,3)$, $(1,4)$, $(2,3)$ or $(2,4)$ we obtain a generically regular network. The generic index of this network is two, regardless of wether we had used one additional resistor, inductor or capacitor branch. In case of an inductor branch the matrix pencil of the resulting network would have two instead of one 2×2 Jordan block associated with a characteristic root at infinity. On the other hand, it is possible to obtain a generically singular index one network using a single additional resistor branch only. Such a branch has to connect one pair of nodes $(3,4)$, $(3,6)$ or $(4,5)$ of $\mathcal{N}$. Observe that the last two cases yield networks with a more degenerated structur, where the

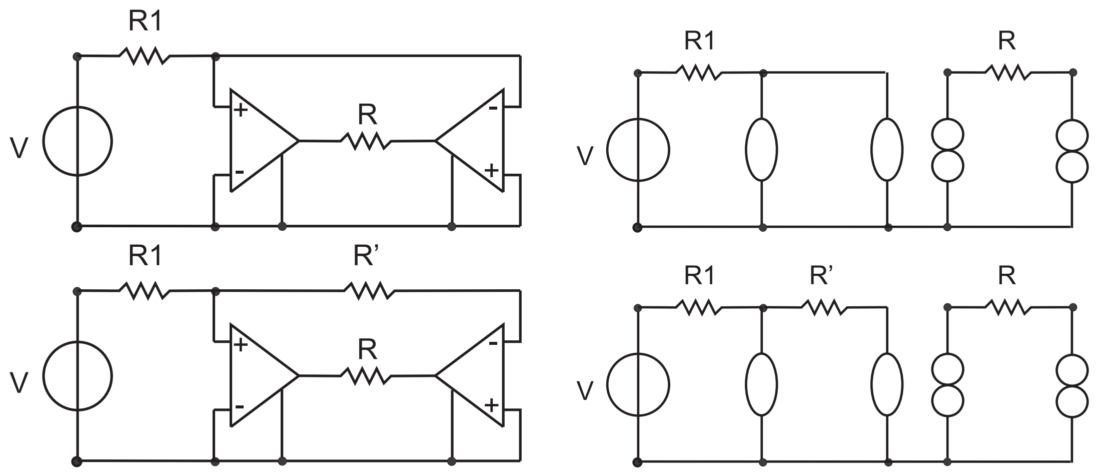


Fig. 3. (a) Original comparator circuit, (b) Network $\mathcal{N}$, (c) Modified circuit, (d) Network $\mathcal{N}_2$

associated matrix pencils contain a singular L_1-Kronecker block, comp. [5,14]. Moreover, augmenting $\mathcal{N}$ with a resistor branch in series to an arbitrary existing branch changes neither the generic singularity nor the generic index.

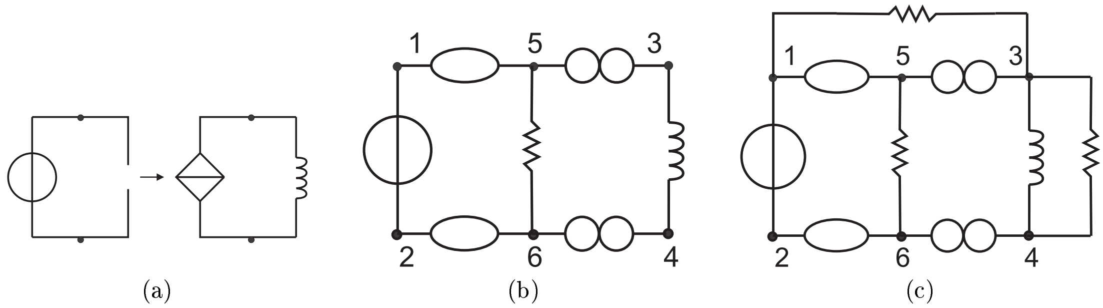


Fig. 4. (a) Original network, (b) Network $\mathcal{N}$, (c) Modified network $\mathcal{N}_2$

Conclusions

A systematic method of augmentation that yields generically regular index one networks has been given. For a class of networks our algorithm uses the minimum number of additional elements possible. The result of this paper is applicable to linear networks that may contain resistors, capacitors, inductors, independend voltage and current sources, and nullator-norator pairs.

Our algorithm is expressed in terms of network graphs. The number of operations to be performed is bounded by a polynomial in the number of network branches.

Further research should be devoted to extend our result to networks containing controlled sources, ideal transformators and gyrators. Moreover, methods for an index two regularization should be derived.

References

1. E. Griepentrog and R. März. *Differential-Algebraic Equations and Their Numerical Treatment*, volume 88 of *Teubner-Texte zur Mathematik*. Teubner Verlagsgesellschaft, Leipzig, 1986.
2. K. E. Brenan, S. L. Campbell, and L. R. Petzold. *Numerical Solution of Initial-Value Problems in Differential-Algebraic Equations*. North-Holland, 1989.
3. E. Hairer, C. Lubich, and M. Roche. *The Numerical Solution of Differential-Algebraic Systems by Runge-Kutta Methods*, volume 1409 of *Lecture Notes in Mathematics*. Springer-Verlag, 1989.
4. G. Reißig. An extension of the normal tree method. In *Proc. Europ. Conf. Circ. Th. Design (ECCTD), Budapest, Hu., Aug. 30–Sept. 3*, 1997.
5. F. R. Gantmacher. *Theory of Matrices*, volume II. Chelsea, New York, 1959.
6. K. Weierstrass. Zur Theorie der bilinearen und quadratischen Formen. In *Monatsbericht der Preussischen Akademie der Wissenschaften*, Berlin, 1868. Reprinted in: Mathematische Werke von Karl Weierstrass, Band II, Mayer & Müller, Berlin, 1895, pages 310-338.
7. K. Röbenack and K. J. Reinschke. Digraph based determination of Jordan block size structure of singular matrix pencils. *Linear Algebra and its Applications*, In print.
8. M. Hasler. Non-linear non-reciprocal resistive circuits with a structurally unique solution. *Int. J. Cir. Theo. Appl.*, 14:237–262, 1986.
9. M. Fosséprez. *Non-linear Circuits - Qualitative Analysis of Non-linear, Non-reciprocal Circuits*. Wiley, 1992.
10. G. Reißig and K. Röbenack. Eine neue Methode zum Auffinden von Paaren konjugierter Bäume. In *Kleinheubacher Berichte 1996, Band 40*, pages 559–565. Deutsche Telekom, 1997.
11. U. Feldmann, M. Hasler, and W. Schwarz. On the design of a synchronizing inverse of a chaotic system. In *Proc. Europ. Conf. Circ. Th. Design (ECCTD) , Istanbul, Turkey*, volume 1, pages 479–482, 1995.
12. Operational Amplifiers and Comparators, Data Book, vol. B. Texas Instruments, 1995.
13. G. Klasche, R. Hofer, D. Nührmann, and H. Pelka. *Professionelle Schaltungstechnik*, volume 2. Franzis Verlag, München, 1993.
14. L. Kronecker. Algebraische Reduktion der Schaaren bilinearer Formen. *Sitzungsberichte der Preussischen Akademie der Wissenschaften*, pages 1225–1237, 1890.